1.7 Find Perimeter, Circumference, and Area

Before You classified polygons.

Now You will find dimensions of polygons.

Why? So you can use measures in science, as in Ex. 46.

Key Vocabulary

- **perimeter,** *p. 923*
- **circumference,** *p. 923*
- **area,** *p. 923*
- **diameter,** *p. 923*
- **radius,** *p. 923*

Recall that *perimeter* is the distance around a figure, *circumference* is the distance around a circle, and *area* is the amount of surface covered by a figure. Perimeter and circumference are measured in units of length, such as meters (m) and feet (ft). Area is measured in square units, such as square meters (m^2) and square feet (ft^2).

KEY CONCEPT *For Your Notebook*

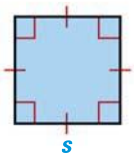
Formulas for Perimeter *P*, Area *A*, and Circumference *C*

Square

side length s

$P = 4s$

$A = s^2$

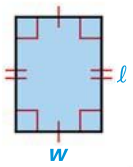


Rectangle

length ℓ and width w

$P = 2\ell + 2w$

$A = \ell w$

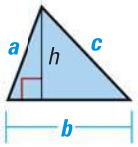


Triangle

side lengths a, b, and c, base b, and height h

$P = a + b + c$

$A = \frac{1}{2}bh$

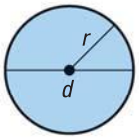


Circle

diameter d and radius r

$C = \pi d = 2\pi r$

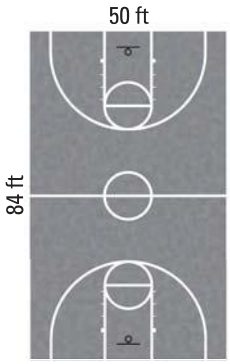
$A = \pi r^2$



Pi (π) is the ratio of a circle's circumference to its diameter.

EXAMPLE 1 Find the perimeter and area of a rectangle

BASKETBALL **Find the perimeter and area of the rectangular basketball court shown.**



Perimeter	Area
$P = 2\ell + 2w$	$A = \ell w$
$= 2(84) + 2(50)$	$= 84(50)$
$= 268$	$= 4200$

▶ The perimeter is 268 feet and the area is 4200 square feet.

EXAMPLE 2 Find the circumference and area of a circle

TEAM PATCH You are ordering circular cloth patches for your soccer team's uniforms. Find the approximate circumference and area of the patch shown.

Solution

First find the radius. The diameter is 9 centimeters, so the radius is $\frac{1}{2}(9) = 4.5$ centimeters.

Then find the circumference and area. Use 3.14 to approximate the value of π.

$$C = 2\pi r \approx 2(3.14)(4.5) = 28.26$$

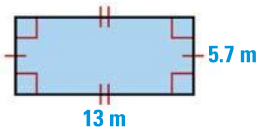
$$A = \pi r^2 \approx 3.14(4.5)^2 = 63.585$$

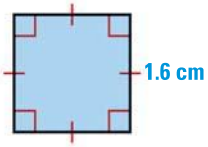
▶ The circumference is about 28.3 cm. The area is about 63.6 cm^2.

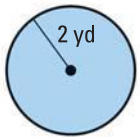
APPROXIMATE π
The approximations 3.14 and $\frac{22}{7}$ are commonly used as approximations for the irrational number π. Unless told otherwise, use 3.14 for π.

GUIDED PRACTICE for Examples 1 and 2

Find the area and perimeter (or circumference) of the figure. If necessary, round to the nearest tenth.

1. 

2. 

3. 

EXAMPLE 3 Standardized Test Practice

Triangle *QRS* has vertices *Q*(1, 2), *R*(4, 6), and *S*(5, 2). What is the approximate perimeter of triangle *QRS*?

Ⓐ 8 units Ⓑ 8.3 units Ⓒ 13.1 units Ⓓ 25.4 units

Solution

First draw triangle *QRS* in a coordinate plane. Find the side lengths. Use the Distance Formula to find *QR* and *RS*.

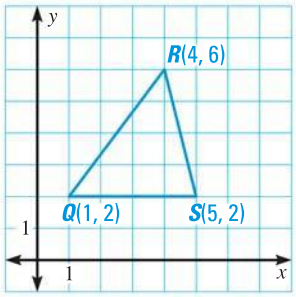
$$QS = |5 - 1| = 4 \text{ units}$$

$$QR = \sqrt{(4-1)^2 + (6-2)^2} = \sqrt{25} = 5 \text{ units}$$

$$RS = \sqrt{(5-4)^2 + (2-6)^2} = \sqrt{17} \approx 4.1 \text{ units}$$

Then find the perimeter.

$$P = QS + QR + RS \approx 4 + 5 + 4.1 = 13.1 \text{ units}$$



▶ The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

AVOID ERRORS
Write down your calculations to make sure you do not make a mistake substituting values in the Distance Formula.

EXAMPLE 4 Solve a multi-step problem

SKATING RINK An ice-resurfacing machine is used to smooth the surface of the ice at a skating rink. The machine can resurface about 270 square yards of ice in one minute.

About how many minutes does it take the machine to resurface a rectangular skating rink that is 200 feet long and 90 feet wide?

ANOTHER WAY
For an alternative method for solving the problem in Example 4, turn to page 57 for the **Problem Solving Workshop**.

Solution

The machine can resurface the ice at a rate of 270 square yards per minute. So, the amount of time it takes to resurface the skating rink depends on its area.

STEP 1 **Find** the area of the rectangular skating rink.

$$\text{Area} = \ell w = 200(90) = 18{,}000 \text{ ft}^2$$

The resurfacing rate is in square yards per minute. Rewrite the area of the rink in square yards. There are 3 feet in 1 yard, and $3^2 = 9$ square feet in 1 square yard.

$$18{,}000 \text{ ft}^2 \cdot \frac{1 \text{ yd}^2}{9 \text{ ft}^2} = 2000 \text{ yd}^2$$ Use unit analysis.

STEP 2 **Write** a verbal model to represent the situation. Then write and solve an equation based on the verbal model.

Let t represent the total time (in minutes) needed to resurface the skating rink.

Area of rink (yd^2)	=	Resurfacing rate (yd^2 per min)	×	Total time (min)

$$2000 = 270 \cdot t$$ Substitute.

$$7.4 \approx t$$ Divide each side by 270.

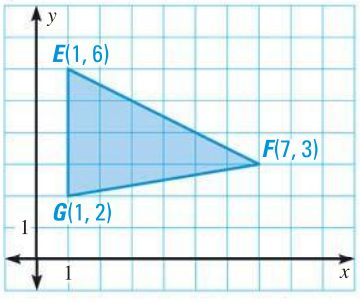
▶ It takes the ice-resurfacing machine about 7 minutes to resurface the skating rink.

GUIDED PRACTICE for Examples 3 and 4

4. Describe how to find the height from F to $\overline{EG}$ in the triangle at the right.

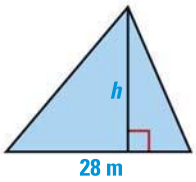
5. Find the perimeter and the area of the triangle shown at the right.

6. **WHAT IF?** In Example 4, suppose the skating rink is twice as long and twice as wide. Will it take an ice-resurfacing machine twice as long to resurface the skating rink? *Explain* your reasoning.



EXAMPLE 5 Find unknown length

The base of a triangle is 28 meters. Its area is 308 square meters. Find the height of the triangle.



Solution

$A = \frac{1}{2}bh$ Write formula for the area of a triangle.

$308 = \frac{1}{2}(28)h$ Substitute 308 for *A* and 28 for *b*.

$22 = h$ Solve for *h*.

▶ The height is 22 meters.

✓ GUIDED PRACTICE for Example 5

7. The area of a triangle is 64 square meters, and its height is 16 meters. Find the length of its base.

1.7 EXERCISES

HOMEWORK KEY

- ○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 21, and 41
- ★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 19, 26, 38, and 45
- ◆ = **MULTIPLE REPRESENTATIONS** Ex. 44

SKILL PRACTICE

1. VOCABULARY How are the diameter and radius of a circle related?

2. ★ WRITING *Describe* a real-world situation in which you would need to find a perimeter, and a situation in which you would need to find an area. What measurement units would you use in each situation?

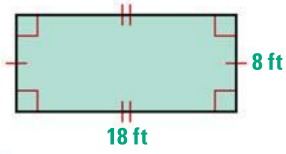
EXAMPLE 1 on p. 49 for Exs. 3–10

3. ERROR ANALYSIS *Describe* and correct the error made in finding the area of a triangle with a height of 9 feet and a base of 52 feet.

$A = 52(9) = 468 \text{ ft}^2$ ✗

PERIMETER AND AREA **Find the perimeter and area of the shaded figure.**

4.



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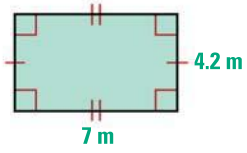

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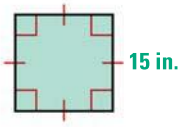

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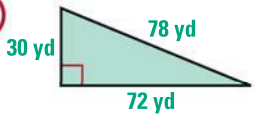

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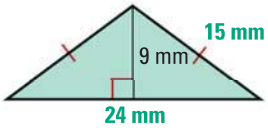

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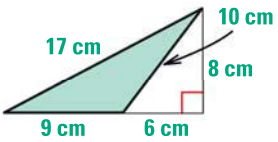


Animated Geometry at classzone.com

10. DRAWING A DIAGRAM The base of a triangle is 32 feet. Its height is $16\frac{1}{2}$ feet. Sketch the triangle and find its area.

EXAMPLE 2 on p. 50 for Exs. 11–15

CIRCUMFERENCE AND AREA **Use the given diameter d or radius r to find the circumference and area of the circle. Round to the nearest tenth.**

11. $d = 27$ cm

12. $d = 5$ in.

13. $r = 12.1$ cm

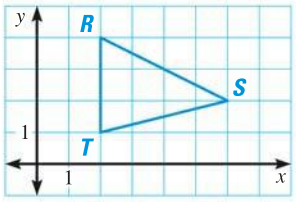
14. $r = 3.9$ cm

15. DRAWING A DIAGRAM The diameter of a circle is 18.9 centimeters. Sketch the circle and find its circumference and area. Round your answers to the nearest tenth.

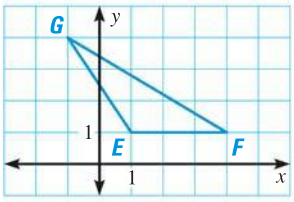
EXAMPLE 3 on p. 50 for Exs. 16–19

DISTANCE FORMULA **Find the perimeter of the figure. Round to the nearest tenth of a unit.**

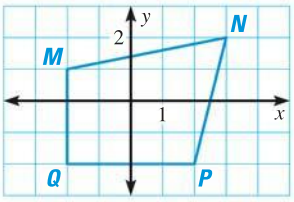
16.



17.



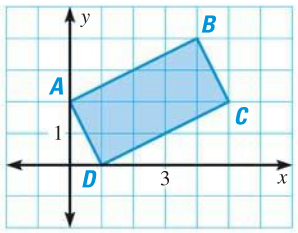
18.



19. ★ MULTIPLE CHOICE What is the approximate area (in square units) of the rectangle shown at the right?

Ⓐ 6.7 Ⓑ 8.0

Ⓒ 9.0 Ⓓ 10.0



EXAMPLE 4 on p. 51 for Exs. 20–26

CONVERTING UNITS **Copy and complete the statement.**

20. 187 cm^2 = ? m^2 **21.** 13 ft^2 = ? yd^2 **22.** 18 $in.^2$ = ? ft^2

23. 8 km^2 = ? m^2 **24.** 12 yd^2 = ? ft^2 **25.** 24 ft^2 = ? $in.^2$

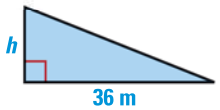
26. ★ MULTIPLE CHOICE A triangle has an area of 2.25 square feet. What is the area of the triangle in square inches?

Ⓐ 27 $in.^2$ Ⓑ 54 $in.^2$ Ⓒ 144 $in.^2$ Ⓓ 324 $in.^2$

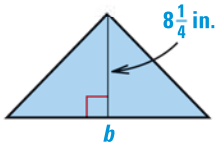
EXAMPLE 5 on p. 52 for Exs. 27–30

UNKNOWN MEASURES **Use the information about the figure to find the indicated measure.**

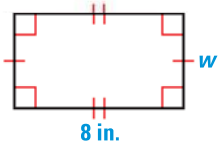
27. Area = 261 m^2
Find the height h.



28. Area = 66 $in.^2$
Find the base b.



29. Perimeter = 25 in.
Find the width w.



30. UNKNOWN MEASURE The width of a rectangle is 17 inches. Its perimeter is 102 inches. Find the length of the rectangle.

31. ALGEBRA The area of a rectangle is 18 square inches. The length of the rectangle is twice its width. Find the length and width of the rectangle.

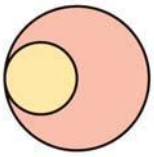
32. ALGEBRA The area of a triangle is 27 square feet. Its height is three times the length of its base. Find the height and base of the triangle.

33. ALGEBRA Let x represent the side length of a square. Find a regular polygon with side length x whose perimeter is twice the perimeter of the square. Find a regular polygon with side length x whose perimeter is three times the length of the square. *Explain* your thinking.

FINDING SIDE LENGTHS **Find the side length of the square with the given area. Write your answer as a radical in simplest form.**

34. $A = 184 \text{ cm}^2$ **35.** $A = 346 \text{ in.}^2$ **36.** $A = 1008 \text{ mi}^2$ **37.** $A = 1050 \text{ km}^2$

38. ★ SHORT RESPONSE In the diagram, the diameter of the yellow circle is half the diameter of the red circle. What fraction of the area of the red circle is *not* covered by the yellow circle? *Explain*.



39. CHALLENGE The area of a rectangle is 30 cm^2 and its perimeter is 26 cm. Find the length and width of the rectangle.

PROBLEM SOLVING

EXAMPLES 1 and 2 on pp. 49–50 for Exs. 40–41

40. WATER LILIES The giant Amazon water lily has a lily pad that is shaped like a circle. Find the circumference and area of a lily pad with a diameter of 60 inches. Round your answers to the nearest tenth.

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41. LAND You are planting grass on a rectangular plot of land. You are also building a fence around the edge of the plot. The plot is 45 yards long and 30 yards wide. How much area do you need to cover with grass seed? How many feet of fencing do you need?

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EXAMPLE 4 on p. 51 for Ex. 42

42. MULTI-STEP PROBLEM Chris is installing a solar panel. The maximum amount of power the solar panel can generate in a day depends in part on its area. On a sunny day in the city where Chris lives, each square meter of the panel can generate up to 125 watts of power. The flat rectangular panel is 84 centimeters long and 54 centimeters wide.

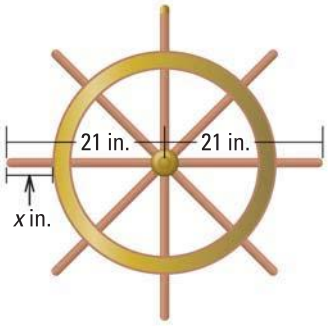
a. Find the area of the solar panel in square meters.

b. What is the maximum amount of power (in watts) that the panel could generate if its area was 1 square meter? 2 square meters? *Explain*.

c. Estimate the maximum amount of power Chris's solar panel can generate. *Explain* your reasoning.

○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE = MULTIPLE REPRESENTATIONS

43. MULTI-STEP PROBLEM The eight spokes of a ship's wheel are joined at the wheel's center and pass through a large wooden circle, forming handles on the outside of the circle. From the wheel's center to the tip of the handle, each spoke is 21 inches long.



a. The circumference of the outer edge of the large wooden circle is 94 inches. Find the radius of the outer edge of the circle to the nearest inch.

b. Find the length x of a handle on the wheel. *Explain*.

44. MULTIPLE REPRESENTATIONS Let x represent the length of a side of a square. Let y_1 and y_2 represent the perimeter and area of that square.

a. Making a Table Copy and complete the table.

Length, x	1	2	5	10	25
Perimeter, y_1	?	?	?	?	?
Area, y_2	?	?	?	?	?

b. Making a Graph Use the completed table to write two sets of ordered pairs: (x, y_1) and (x, y_2). Graph each set of ordered pairs.

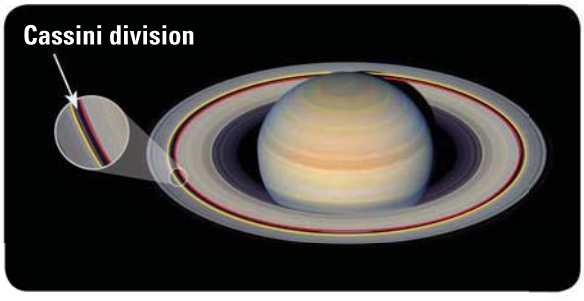
c. Analyzing Data *Describe* any patterns you see in the table from part (a) and in the graphs from part (b).

45. ★ EXTENDED RESPONSE The photograph at the right shows the Crown Fountain in Chicago, Illinois. At this fountain, images of faces appear on a large screen. The images are created by light-emitting diodes (LEDs) that are clustered in groups called modules. The LED modules are arranged in a rectangular grid.

a. The rectangular grid is approximately 7 meters wide and 15.2 meters high. Find the area of the grid.

b. Suppose an LED module is a square with a side length of 4 centimeters. How many rows and how many columns of LED modules would be needed to make the Crown Fountain screen? *Explain* your reasoning.

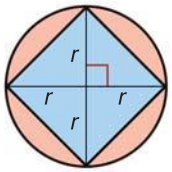
46. ASTRONOMY The diagram shows a gap in Saturn's circular rings. This gap is known as the *Cassini division*. In the diagram, the red circle represents the ring that borders the inside of the Cassini division. The yellow circle represents the ring that borders the outside of the division.



a. The radius of the red ring is 115,800 kilometers. The radius of the yellow ring is 120,600 kilometers. Find the circumference of the red ring and the circumference of the yellow ring. Round your answers to the nearest hundred kilometers.

b. Compare the circumferences of the two rings. About how many kilometers greater is the yellow ring's circumference than the red ring's circumference?

47. CHALLENGE In the diagram at the right, how many times as great is the area of the circle as the area of the square? *Explain* your reasoning.



48. ALGEBRA You have 30 yards of fencing with which to make a rectangular pen. Let x be the length of the pen.

a. Write an expression for the width of the pen in terms of x. Then write a formula for the area y of the pen in terms of x.

b. You want the pen to have the greatest possible area. What length and width should you use? *Explain* your reasoning.

MIXED REVIEW

PREVIEW
Prepare for Lesson 2.1 in Exs. 49–50.

49. Use the equation $y = 2x + 1$ to copy and complete the table of values. *(p. 884)*

x	1	2	3	4	5
y	?	?	?	?	?

50. Each number in a pattern is 6 less than the previous number. The first number in the pattern is 100. Write the next three numbers. *(p. 894)*

In Exercises 51 and 52, draw a diagram to represent the problem. Then find the indicated measure. *(p. 42)*

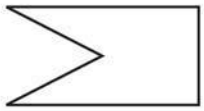
51. The lengths (in inches) of two sides of a regular triangle are given by the expressions $5x + 40$ and $8x - 13$. Find the length of a side of the triangle.

52. The measures of two angles of an equiangular hexagon are $12x°$ and $(10x + 20)°$. Find the measure of an angle of the hexagon.

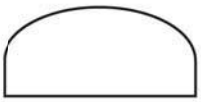
QUIZ *for Lessons 1.6–1.7*

Tell whether the figure is a polygon. If it is not, *explain* why. If it is a polygon, tell whether it is *convex* or *concave*. *(p. 42)*

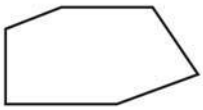
1.



2.

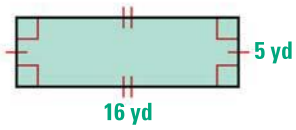


3.

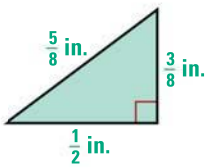


Find the perimeter and area of the shaded figure. *(p. 49)*

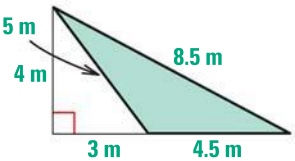
4.



5.



6.



7. GARDENING You are spreading wood chips on a rectangular garden. The garden is $3\frac{1}{2}$ yards long and $2\frac{1}{2}$ yards wide. One bag of wood chips covers 10 square feet. How many bags of wood chips do you need? *(p. 49)*